Advances in the measurement of the Lense-Thirring effect with Satellite Laser Ranging in the gravitational field of the Earth

Lorenzo Iorio

INFN-Sezione di Pisa. Permanent address for correspondence: Viale Unità di Italia 68, 70125, Bari (BA), Italy. E-mail: lorenzo.iorio@libero.it

ABSTRACT

In this Chapter we deal with the attempts to measure the general relativistic gravitomagnetic Lense-Thirring effect with the Satellite Laser Ranging (SLR) technique applied to the existing LAGEOS and LAGEOS II terrestrial satellites and to the recently approved LARES. According to general relativity, the rotation of the central body which acts as source of the gravitational field changes the spatial orientation of the orbit of test particles determined by the longitude of the ascending node Ω. Such a shift, which is cumulative in time, is of the order of 2 m yr^{-1} in the case of the LAGEOS satellites. Extracting this signature from the data is a demanding task because of many classical orbital perturbations having the same pattern as the gravitomagnetic one, like those induced by the centrifugal oblateness of the Earth which represents a major source of systematic error. The first issue addressed here is: are the so far published evaluations of the systematic bias induced by the uncertainty in the even zonal harmonic coefficients J_ℓ of the multipolar expansion of the Earth's geopotential reliable and realistic? The answer is negative. Indeed, if the difference ΔJ_ℓ among the even zonals estimated in different global solutions (EIGEN-GRACE02S, EIGEN-CG03C, GGM02S, GGM03S, ITG-Grace02s, ITG-Grace03s, JEM01-RL03B, EGM2008) is assumed for the uncertainties δJ_ℓ instead of using their more or less calibrated covariance sigmas σ_{J_ℓ}, it turns out that the systematic error $\delta\mu$ in the Lense-Thirring test with the nodes of LAGEOS and LAGEOS II is about 3 to 4 times larger than in the evaluations so far published based on the use of the sigmas of one model at a time separately, amounting up to 37% for the pair EIGEN-GRACE02S/ITG-Grace03s. The comparison among the other recent GRACE-based models yields bias as large as about $25 - 30\%$. The major discrepancies still occur for J_4, J_6 and J_8, which are just the zonals the combined

LAGEOS/LAGOES II nodes are most sensitive to. The second issue is the possibility of reaching a realistic total accuracy of 1% with LAGEOS, LAGEOS II and LARES, which will be launched in the near future. While LAGEOS and LAGEOS II fly at altitudes of about 6000 km, LARES will be likely placed at an altitude of 1200 km. Thus, it will be sensitive to much more even zonals than LAGEOS and LAGEOS II. Their corrupting impact has been evaluated with the standard Kaula's approach up to degree $\ell = 60$ by using ΔJ_ℓ; it turns out that it may be as large as some tens percent. The different orbit of LARES may also have some consequences on the non-gravitational orbital perturbations affecting it which might further degrade the obtainable accuracy.

1. Introduction

In the weak-field and slow motion approximation, valid when the magnitude of the gravitational potentials U and velocities v characteristic of the problem under examination are smaller with respect to the speed of light c, i.e. for $U/c^2, v/c \lll 1$, the Einstein field equations of general relativity get linearized resembling to the Maxwellian equations of electromagntism. As a consequence, a gravitomagnetic field, induced by the off-diagonal components $g_{0i}, i = 1, 2, 3$ of the space-time metric tensor related to the mass-energy currents of the source of the gravitational field, arises (Mashhoon 2007); it has no classical counterparts in Newtonian mechanics. The gravitomagnetic field affects orbiting test particles, precessing gyroscopes, moving clocks and atoms and propagating electromagnetic waves (Ruggiero and Tartaglia 2002; Schäfer 2004). Perhaps, the most famous gravitomagnetic effects are the precession of the axis of a gyroscope (Pugh 1959;

Schiff 1960) and the Lense-Thirring[1] precessions (Lense and Thirring 1918) of the orbit
of a test particle, both occurring in the field of a central slowly rotating mass like, e.g.,
our planet. Direct, undisputable measurements of such fundamental predictions of general
relativity are not yet available.

The measurement of the gyroscope precession in the Earth's gravitational field has
been the goal of the dedicated space-based[2] GP-B mission (Everitt et al 1974, 2001)
launched in 2004 and carrying onboard four superconducting gyroscopes; its data analysis
is still ongoing. The target accuracy was originally 1%, but it is still unclear if the GP-B
team will succeed in reaching such a goal because of some unmodelled effects affecting the
gyroscopes: 1) a time variation in the polhode motion of the gyroscopes and 2) very large
classical misalignment torques on the gyroscopes.

In this paper we will focus on the measurement of the Lense-Thirring effect in the
gravitational field of the Earth. It consists of a secular rate of the longitude of the ascending
node Ω

$$\dot{\Omega}_{\mathrm{LT}} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{3/2}}, \tag{1}$$

and of the argument of pericentre ω

$$\dot{\omega}_{\mathrm{LT}} = -\frac{6GJ \cos i}{c^2 a^3 (1 - e^2)^{3/2}}, \tag{2}$$

of the orbit of a test particle. In eq. (1) and eq. (2) G is the Newtonian constant of
gravitation, J is the proper angular momentum of the central body, a and e are the
semimajor axis and the eccentricity, respectively, of the test particle's orbit and i is its
inclination to the central body's equator. The semimajor axis a determines the size of the

---

[1]According to an interesting historical analysis recently performed in (Pfister 2007), it
would be more correct to speak about an Einstein-Thirring-Lense effect.

[2]See on the WEB http://einstein.stanford.edu/

ellipse, while its shape is controlled by the eccentricity $0 \leq e < 1$; an orbit with $e = 0$ is a circle. The angles Ω and ω fix the orientation of the orbit in the inertial space and in the orbital plane, respectively. Ω, ω and i can be viewed as the three Euler angles which determine the orientation of a rigid body with respect to an inertial frame. In Figure 1 we illustrate the geometry of a Keplerian orbit.

In this Chapter we will critically discuss the following two topics

- The realistic evaluation in Section 2 of the total accuracy in the test performed in recent years with the existing Earth's artificial satellites LAGEOS and LAGEOS II (Ciufolini and Pavlis 2004; Ciufolini et al 2006). LAGEOS was put into orbit in 1976, followed by its twin LAGEOS II in 1992; they are passive, spherical spacecraft entirely covered by retroreflectors which allow for their accurate tracking through laser pulses sent from Earth-based ground stations according to the Satellite Laser Ranging (SLR) technique. They orbit at altitudes of about 6000 km ($a_{\mathrm{LAGEOS}} = 12270$ km, $a_{\mathrm{LAGEOS\ II}} = 12163$ km) in nearly circular paths ($e_{\mathrm{LAGEOS}} = 0.0045$, $e_{\mathrm{LAGEOS\ II}} = 0.014$) inclined by 110 deg and 52.65 deg, respectively, to the Earth's equator. The Lense-Thirring effect for their nodes amounts to about 30 milliarcseconds per year (mas yr^{-1}) which correspond to about 2 m yr^{-1} at the LAGEOS altitudes.

  The idea of measuring the Lense-Thirring node rate with the just launched LAGEOS satellite, along with the other SLR targets orbiting at that time, was put forth by Cugusi and Proverbio (1978). Tests have started to be effectively performed later by using the LAGEOS and LAGEOS II satellites (Ciufolini et al 1996), according to a strategy (Ciufolini 1996) involving the use of a suitable linear combination of the nodes Ω of both satellites and the perigee ω of LAGEOS II. This was done to reduce the impact of the most relevant source of systematic bias, i.e. the mismodelling in
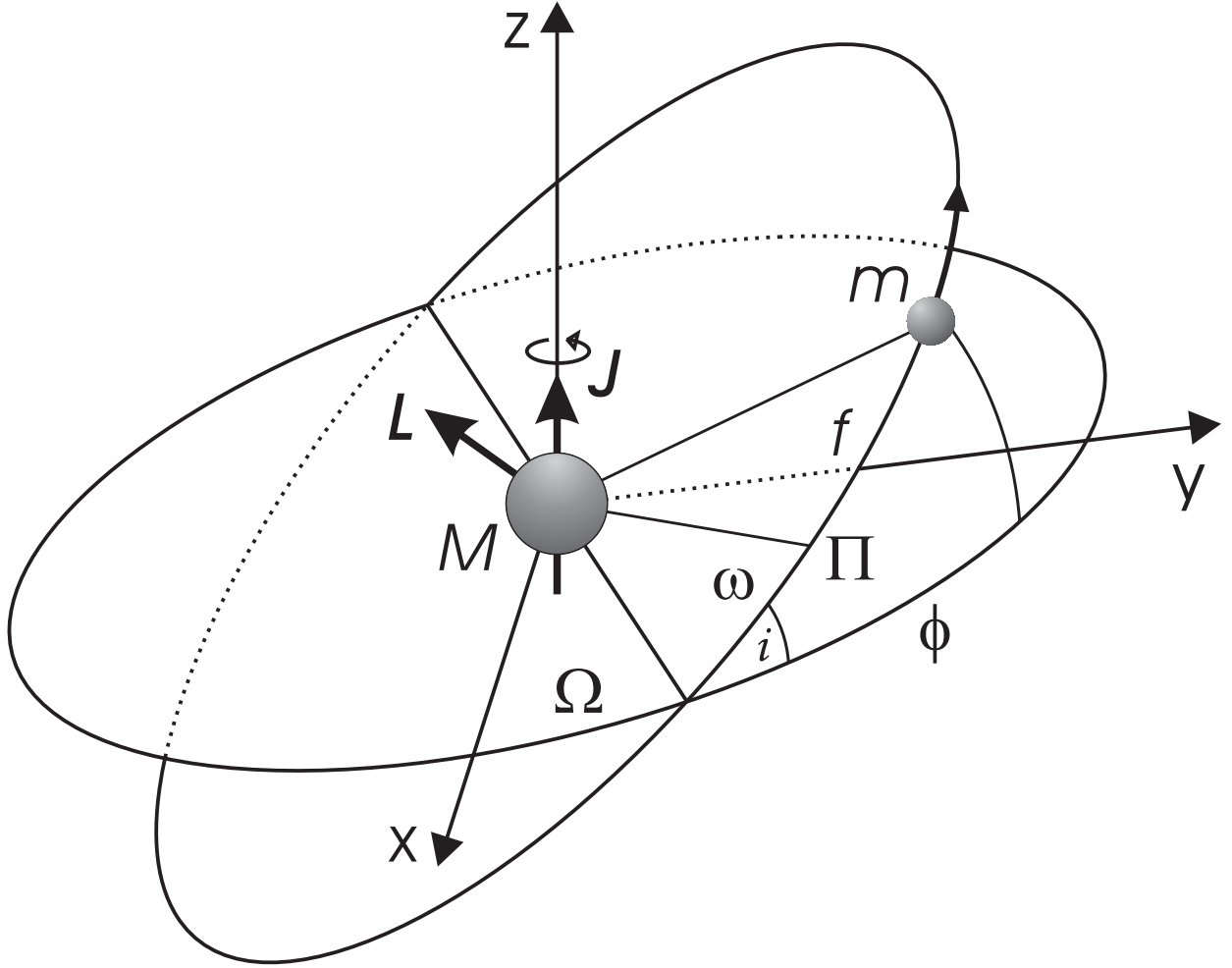
Fig. 1.— Keplerian orbit. The longitude of the ascending node Ω is counted from a reference X direction in the reference {XY} plane, chosen coincident with the equatorial plane of the rotating body of mass M and proper angular momentum J, to the line of the nodes, i.e. the intersection of the orbital plane with the reference plane. The argument of pericentre ω is an angle in the orbital plane counted from the line of the nodes; the location of the pericentre is marked with Π. The time-dependent position of the moving test particle of mass m is given by the true anomaly f, counted anticlockwise from the pericentre; ϕ is an azimuthal angle in the {XY} plane. L is the orbital angular momentum, perpendicular to the orbital plane. The inclination between the orbital plane and the equatorial plane of M is i. Courtesy by H. Lichtenegger, OEAW, Graz.

the even ($\ell = 2, 4, 6 \ldots$) zonal ($m = 0$) harmonic coefficients J_ℓ of the multipolar expansion of the Newtonian part of the terrestrial gravitational potential due to the diurnal rotation (they induce secular precessions on the node and perigee of a terrestrial satellite much larger than the gravitomagnetic ones. The J_ℓ coefficients cannot be theoretically computed but must be measured by using dedicated satellites; see Section 2): the three-elements combination used allowed for removing the uncertainties in J_2 and J_4. In (Ciufolini et al 1998b) a $\approx 20\%$ test was reported by using the[3] EGM96 (Lemoine et al 1998) Earth gravity model; subsequent detailed analyses showed that such an evaluation of the total error budget was overly optimistic in view of the likely unreliable computation of the total bias due to the even zonals (Iorio 2003; Ries et al 2003a,b). An analogous, huge underestimation turned out to hold also for the effect of the non-gravitational perturbations (Milani et al 1987) like the direct solar radiation pressure, the Earth's albedo, various subtle thermal effects depending on the physical properties of the satellites' surfaces and their rotational state (Inversi and Vespe 1994; Vespe 1999; Lucchesi 2001; Lucchesi 2002, 2003, 2004; Lucchesi et al 2004; Ries et al 2003a), which the perigees of LAGEOS-like satellites are particularly sensitive to. As a result, the realistic total error budget in the test reported in (Ciufolini et al 1998b) might be as large as $60 - 90\%$ or even more (by considering EGM96 only).

---

[3]Contrary to the subsequent models based on the dedicated satellites CHAMP (http://www-app2.gfz-potsdam.de/pb1/op/champ/index_CHAMP.html) and GRACE (http://www-app2.gfz-potsdam.de/pb1/op/grace/index_GRACE.html), EGM96 relies upon multidecadal tracking of SLR data of a constellation of geodetic satellites including LAGEOS and LAGEOS II as well; thus the possibility of a sort of $a - priori$ 'imprinting' of the Lense-Thirring effect itself, not solved-for in EGM96, cannot be neglected.

The observable used in (Ciufolini and Pavlis 2004) with the GRACE-only EIGEN-GRACE02S model (Reigber et al 2005) was the following linear combination[4] of the nodes of LAGEOS and LAGEOS II, explicitly computed in (Iorio and Morea 2004) following the approach put forth in (Ciufolini 1996)

$$f = \dot{\Omega}^{\text{LAGEOS}} + c_1 \dot{\Omega}^{\text{LAGEOS II}}, \tag{3}$$

where

$$c_1 \equiv -\frac{\dot{\Omega}_{.2}^{\text{LAGEOS}}}{\dot{\Omega}_{.2}^{\text{LAGEOS II}}} = -\frac{\cos i_{\text{LAGEOS}}}{\cos i_{\text{LAGEOS II}}} \left(\frac{1 - e_{\text{LAGEOS II}}^2}{1 - e_{\text{LAGEOS}}^2}\right)^2 \left(\frac{a_{\text{LAGEOS II}}}{a_{\text{LAGEOS}}}\right)^{7/2}. \tag{4}$$

The coefficients $\dot{\Omega}_{.\ell}$ of the aliasing classical node precessions (Kaula 1966) $\dot{\Omega}_{\text{class}} = \sum_\ell \dot{\Omega}_{.\ell} J_\ell$ induced by the even zonals have been analytically worked out in, e.g. (Iorio 2003); they yield $c_1 = 0.544$. The Lense-Thirring signature of eq. (3) amounts to 47.8 mas yr^{-1}. The combination eq. (3) allows, by construction, to remove the aliasing effects due to the static and time-varying parts of the first even zonal J_2. The nominal (i.e. computed with the estimated values of J_ℓ, $\ell = 4, 6...$) bias due to the remaining higher degree even zonals would amount to about 10^5 mas yr^{-1}; the need of a careful and reliable modeling of such an important source of systematic bias is, thus, quite apparent. Conversely, the nodes of the LAGEOS-type spacecraft are affected by the non-gravitational accelerations at a $\approx 1\%$ level of the Lense-Thirring effect (Lucchesi 2001; Lucchesi 2002, 2003, 2004; Lucchesi et al 2004). For a comprehensive, up-to-date overview of the numerous and subtle issues concerning the measurement of the Lense-Thirring effect see (Iorio 2007d).

- The possibility that the LARES mission, recently approved by the Italian Space Agency (ASI), will be able to measure the Lense-Thirring node precession with an accuracy of the order of 1% (Section 3).

---

[4]See also (Pavlis 2002; Ries et al 2003a,b).

In (Van Patten and Everitt 1976a; van Patten and Everitt 1976b) it was proposed to measure the Lense-Thirring precession of the nodes Ω of a pair of counter-orbiting spacecraft to be launched in terrestrial polar orbits and endowed with drag-free apparatus. A somewhat equivalent, cheaper version of such an idea was put forth in 1986 by (Ciufolini 1986) who proposed to launch a passive, geodetic satellite in an orbit identical to that of LAGEOS apart from the orbital planes which should have been displaced by 180 deg apart. The measurable quantity was, in the case of the proposal by Ciufolini (1986), the sum of the nodes of LAGEOS and of the new spacecraft, later named LAGEOS III, LARES, WEBER-SAT, in order to cancel to a high level of accuracy the corrupting effect of the multipoles of the Newtonian part of the terrestrial gravitational potential which represent the major source of systematic error (see Section 2). Although extensively studied by various groups (Ries et al 1989; Ciufolini et al 1998a), such an idea was not implemented for many years. In (Iorio et al 2002) it was proposed to include also the data from LAGEOS II by using a different observable. Such an approach was proven in (Iorio 2005a) to be potentially useful in making the constraints on the orbital configuration of the new SLR satellite less stringent than it was originally required in view of the recent improvements in our knowledge of the classical part of the terrestrial gravitational potential due to the dedicated CHAMP and, especially, GRACE missions.

Since reaching high altitudes and minimizing the unavoidable orbital injection errors is expensive, it was explored the possibility of discarding LAGEOS and LAGEOS II using a low-altitude, nearly polar orbit for LARES (Lucchesi and Paolozzi 2001; Ciufolini 2006), but in (Iorio 2002, 2007a) it was proven that such alternative approaches are not feasible. It was also suggested that LARES would be able to probe alternative theories of gravity (Ciufolini 2004), but also in this case it turned out to be impossible (Iorio 2005b, 2007b).

The stalemate came to an end when ASI recently made the following official announcement (http://www.asi.it/SiteEN/MotorSearchFullText.aspx?keyw=LARES): "On February 8, the ASI board approved funding for the LARES mission, that will be launched with VEGAs maiden flight before the end of 2008. LARES is a passive satellite with laser mirrors, and will be used to measure the Lense-Thirring effect." The italian version of the announcement yields some more information specifying that LARES, designed in collaboration with National Institute of Nuclear Physics (INFN), is currently under construction by Carlo Gavazzi Space SpA; its Principal Investigator (PI) is I. Ciufolini and its scientific goal is to measure at a 1% level the Lense-Thirring effect in the gravitational field of the Earth. Concerning the orbital configuration of LARES, the ASI website says about VEGA that (http://www.asi.it/SiteEN/ContentSite.aspx?Area=Accesso+allo+spazio): "[...] VEGA can place a 15.000 kg satellite on a low polar orbit, 700 km from the Earth. By lowering the orbit inclination it can launch heavier payloads, whereas diminishing the payload mass it can achieve higher orbits. [...]" In the latest communication to INFN, Rome, 30 January 2008, Ciufolini (2008) writes that LARES will be launched with a semimajor axis of approximately 7600 km and an inclination between 60 and 80 deg. More precise information can be retrieved in Section 5.1, pag 9 of the document Educational Payload on the Vega Maiden Flight Call For CubeSat Proposals, European Space Agency, Issue 1 11 February 2008, downloadable at http://esamultimedia.esa.int/docs/LEX-EC/CubeSat%20CFP%20issue%201.pdf. It is written there that LARES will be launched into a circular orbit with altitude $h = 1200$ km, corresponding to a semimajor axis $a_{\mathrm{LARES}} = 7578$ km, and inclination $i = 71$ deg to the Earth's equator.

2. The systematic error of gravitational origin in the LAGEOS-LAGEOS II test

The realistic evaluation of the total error budget of the LAGEOS-LAGEOS II node test (Ciufolini and Pavlis 2004) raised a lively debate (Ciufolini and Pavlis 2005; Ciufolini et al 2006; Iorio 2005a, 2006a,b, 2007c; Lucchesi 2005), mainly focussed on the impact of the static and time-varying parts of the Newtonian component of the Earth's gravitational potential through the aliasing secular precessions induced on a satellite's node.

In a realistic scenario the path of a probe is not only affected by the gravitomagentic field but also by a huge number of other competing orbital perturbations of gravitational and non-gravitational origin. The most important non-conservative accelerations (Milani et al 1987) are the direct solar radiation pressure, the Earth's albedo and various subtle thermal effects depending on the the physical properties of the satellite's surface and its rotational state (Inversi and Vespe 1994; Vespe 1999; Lucchesi 2001; Lucchesi 2002, 2003, 2004; Lucchesi et al 2004; Ries et al 2003a); however, the nodes of LAGEOS-like satellites are sensitive to them at a $\approx 1\%$ level only. Much more important is the impact that the oblateness of the Earth, due to its diurnal rotation, has on the satellite's dynamics. Indeed, the most insidious perturbations are those induced by the static part of the Newtonian component of the multipolar expansion in spherical harmonics[5] $J_\ell, \ell = 2, 4, 6, ...$ of the gravitational potential of the central rotating mass (Kaula 1966): indeed, they affect the node with effects having the same signature of the relativistic signal of interest, i.e. linear trends which are orders of magnitude larger and cannot be removed from the time series of data without affecting the Lense-Thirring pattern itself as well. The only thing that can be done is to model such a corrupting effect as most accurately as possible and assessing,

---

[5]The relation among the even zonals J_ℓ and the normalized gravity coefficients $\overline{C}_{\ell 0}$ which are customarily determined in the Earth's gravity models, is $J_\ell = -\sqrt{2\ell + 1}\, \overline{C}_{\ell 0}$.

reliably and realistically, the impact of the residual mismodelling on the measurement of the frame-dragging effect. The secular precessions induced by the even zonals of the geopotential can be written as

$$\dot{\Omega}^{\text{geopot}} = \sum_{\ell=2} \dot{\Omega}_{.\ell} J_\ell, \tag{5}$$

where the coefficients $\dot{\Omega}_{.\ell}, \ell = 2, 4, 6, ...$ depend on the parameters of the Earth (GM and the equatorial radius R) and on the semimajor axis a, the inclination i and the eccentricity e of the satellite. For example, for $\ell = 2$ we have

$$\dot{\Omega}_{.2} = -\frac{3}{2}n \left(\frac{R}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}; \tag{6}$$

$n = \sqrt{GM/a^3}$ is the Keplerian mean motion. They have been analytically computed up to $\ell = 20$ in, e.g., (Iorio 2003). Their mismodelling can be written as

$$\delta\dot{\Omega}^{\text{geopot}} \leq \sum_{\ell=2} \left|\dot{\Omega}_{.\ell}\right| \delta J_\ell, \tag{7}$$

where δJ_ℓ represents our uncertainty in the knowledge of the even zonals J_ℓ

A common feature of all the competing evaluations so far published is that the systematic bias due to the static component of the geopotential was calculated always by using the released (more or less accurately calibrated) sigmas σ_{J_ℓ} of one Earth gravity model solution at a time for the uncertainties δJ_ℓ in the even zonal harmonics, so to say that the model X yields a $x\%$ error, the model Y yields a $y\%$ error, and so on.

Since a trustable calibration of the formal, statistical uncertainties in the estimated zonals of the covariance matrix of a global solution is always a difficult task to be implemented in a reliable way, a much more realistic and conservative approach consists, instead, of taking the difference[6] ΔJ_ℓ of the estimated even zonals for different pairs of Earth gravity field solutions as representative of the real uncertainty δJ_ℓ in the zonals

---

[6]See Fig.5 of (Lucchesi 2007) for a comparison of the estimated $\overline{C}_{40}$ in different models.

(Lerch et al 1994). In Table 1–Table 10 we present our results for the most recent GRACE-based models released so far by different institutions and retrievable on the Internet at[7] http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html. The models used are EIGEN-GRACE02S (Reigber et al 2005) and EIGEN-CG03C (Förste et al 2005) from GFZ (Potsdam, Germany), GGM02S (Tapley et al 2005) and GGM03S (Tapley et al 2007) from CSR (Austin, Texas), ITG-Grace02s (Mayer-Gürr et al 2006) and ITG-Grace03s (Mayer-Gürr 2007) from IGG (Bonn, Germany), JEM01-RL03B from JPL (NASA, USA) and EGM2008 (Pavlis et al. 2008) from NGA (USA). Note that this approach was explicitly followed also by Ciufolini (1996) with the JGM3 and GEMT-2 models.

The systematic bias evaluated with a more realistic approach is about 3 to 4 times larger than one can obtain by only using this or that particular model. The scatter is still quite large and far from the $5 - 10\%$ claimed in (Ciufolini and Pavlis 2004). In particular, it appears that J_4, J_6, and to a lesser extent J_8, which are just the most relevant zonals for us because of their impact on the combination of eq. (3), are the most uncertain ones, with discrepancies ΔJ_ℓ between different models, in general, larger than the sum of their sigmas σ_{J_ℓ}, calibrated or not. This is an important feature because the other alternative combinations proposed involving more satellites (Iorio and Doornbos 2005; Vespe and Rutigliano 2005) should be less affected since they cancel out the impact of J_4 and J_6 as well.

Another approach that could be followed to take into account the scatter among the various solutions consists in computing mean and standard deviation of the entire set of values of the even zonals for the models considered so far, degree by degree, and taking the standard deviations as representative of the uncertainties $\delta J_\ell, \ell = 4, 6, 8, ...$ It yields

---

[7]I thank J Ries, CSR, and M Watkins (JPL) for having provided me with the even zonals of the GGM03S (Tapley et al 2007) and JEM01-RL03B models.

$\delta\mu = 15\%$.

It must be recalled that also the further bias due to the cross-coupling between J_2 and the orbit inclination, evaluated to be about 9% in (Iorio 2007c), must be added.

3. A conservative evaluation of the impact of the geopotential on the LARES mission

The combination which will be used for measuring the Lense-Thirring effect with LAGEOS, LAGEOS II and LARES is (Iorio 2005a)

$$\dot{\Omega}^{\text{LAGEOS}} + k_1\dot{\Omega}^{\text{LAGEOS II}} + k_2\dot{\Omega}^{\text{LARES}}. \tag{8}$$

The coefficients k_1 and k_2 entering eq. (8) are defined as

$$k_1 = \frac{\dot{\Omega}^{\text{LARES}}_{.2}\dot{\Omega}^{\text{LAGEOS}}_{.4} - \dot{\Omega}^{\text{LAGEOS}}_{.2}\dot{\Omega}^{\text{LARES}}_{.4}}{\dot{\Omega}^{\text{LAGEOS II}}_{.2}\dot{\Omega}^{\text{LARES}}_{.4} - \dot{\Omega}^{\text{LARES}}_{.2}\dot{\Omega}^{\text{LAGEOS II}}_{.4}} = 0.3697,$$

$$k_2 = \frac{\dot{\Omega}^{\text{LAGEOS}}_{.2}\dot{\Omega}^{\text{LAGEOS II}}_{.4} - \dot{\Omega}^{\text{LAGEOS II}}_{.2}\dot{\Omega}^{\text{LAGEOS}}_{.4}}{\dot{\Omega}^{\text{LAGEOS II}}_{.2}\dot{\Omega}^{\text{LARES}}_{.4} - \dot{\Omega}^{\text{LARES}}_{.2}\dot{\Omega}^{\text{LAGEOS II}}_{.4}} = 0.0619. \tag{9}$$

The combination eq. (8) cancels out, by construction, the impact of the first two even zonals. The total Lense-Thirring effect, according to eq. (8) and eq. (9), amounts to 47.8 mas yr^{-1}. The systematic error due to the uncancelled even zonals $J_6, J_8, ...$ can be conservatively evaluated as

$$\delta\mu \leq \sum_{\ell=6} \left| \dot{\Omega}^{\text{LAGEOS}}_{.\ell} + k_1\dot{\Omega}^{\text{LAGEOS II}}_{.\ell} + k_2\dot{\Omega}^{\text{LARES}}_{.\ell} \right| \delta J_\ell \tag{10}$$

Of crucial importance is how to assess δJ_ℓ. By proceeding as in Section 2 and by using the same models up to degree $\ell = 60$ because of the lower altitude of LARES with respect to LAGEOS and LAGEOS II which brings into play more even zonals, we have the results presented in Table 11.

Table 1: Impact of the mismodelling in the even zonal harmonics on $f_\ell = \left| \dot{\Omega}_\ell^{\text{LAGEOS}} + c_1 \dot{\Omega}_{,\ell}^{\text{LAGEOS II}} \right| \Delta J_\ell,\ \ell = 4, \ldots, 20$, in mas yr^{-1}. Recall that $J_\ell = -\sqrt{2\ell + 1}\ \overline{C}_{\ell 0}$; for the uncertainty in the even zonals we have taken here the difference $\Delta \overline{C}_{\ell 0} = \left| \overline{C}_{\ell 0}^{(\text{X})} - \overline{C}_{\ell 0}^{(\text{Y})} \right|$ between the model X = EIGEN-CG03C (Förste et al 2005) and the model Y = EIGEN-GRACE02S (Reigber et al 2005). EIGEN-CG03C combines data from CHAMP (860 days out of October 2000 to June 2003), GRACE (376 days out of February to May 2003, July to December 2003 and February to July 2004) and terrestrial measurements; EIGEN-GRACE02S is based on 110 days (out of August and November 2002 and April, May and August 2003) of GRACE-only GPS-GRACE high-low satellite-to-satellite data, on-board measurements of non-gravitational accelerations, and especially GRACE intersatellite tracking data. $\sigma_{\text{X/Y}}$ are the covariance calibrated errors for both models. Values of f_ℓ smaller than 0.1 mas yr^{-1} have not been quoted. The Lense-Thirring precession of the combination of eq. (3) amounts to 47.8 mas yr^{-1}. The percent bias $\delta\mu$ has been computed by normalizing the linear sum of $f_\ell, \ell = 4, \ldots, 20$ to the Lense-Thirring precession. The discrepancies between the models are significant since $\Delta \overline{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, \ldots 16$.

ℓ	$\Delta \overline{C}_{\ell 0}$ (EIGEN-CG03C-EIGEN-GRACE02S)	$\sigma_{\text{X}} + \sigma_{\text{Y}}$	f_ℓ (mas yr^{-1})
4	1.96×10^{-11}	1.01×10^{-11}	7.3
6	2.50×10^{-11}	4.8×10^{-12}	5.4
8	4.9×10^{-12}	3.3×10^{-12}	0.2
10	3.7×10^{-12}	3.4×10^{-12}	-
12	2.5×10^{-12}	2.3×10^{-12}	-
14	6.1×10^{-12}	2.1×10^{-12}	-
16	2.1×10^{-12}	1.7×10^{-12}	-
18	6×10^{-13}	1.7×10^{-12}	-
20	1.7×10^{-12}	1.7×10^{-12}	-
	total bias $\delta\mu = 27\%$		

Table 2: Impact of the mismodelling in the even zonal harmonics as solved for in X=GGM02S (Tapley et al 2005) and Y=ITG-Grace02s (Mayer-Gürr et al 2006). GGM02S is based on 363 days of GRACE-only data (GPS and intersatellite tracking, neither constraints nor regularization applied) spread between April 4, 2002 and Dec 31, 2003. The σ are formal for both models. $\Delta \overline{C}_{\ell 0}$ are always larger than the linearly added sigmas, apart from $\ell = 12$ and $\ell = 18$.

ℓ	$\Delta \overline{C}_{\ell 0}$ (GGM02S-ITG-Grace02s)	$\sigma_{\mathrm{X}} + \sigma_{\mathrm{Y}}$	f_{ℓ} (mas yr^{-1})
4	1.9×10^{-11}	8.7×10^{-12}	7.2
6	2.1×10^{-11}	4.6×10^{-12}	4.6
8	5.7×10^{-12}	2.8×10^{-12}	0.2
10	4.5×10^{-12}	2.0×10^{-12}	-
12	1.5×10^{-12}	1.8×10^{-12}	-
14	6.6×10^{-12}	1.6×10^{-12}	-
16	2.9×10^{-12}	1.6×10^{-12}	-
18	1.4×10^{-12}	1.6×10^{-12}	-
20	2.0×10^{-12}	1.6×10^{-12}	-
	total bias $\delta\mu = 25\%$		

Table 3: Impact of the mismodelling in the even zonal harmonics as solved for in X=GGM02S (Tapley et al 2005) and Y=EIGEN-CG03C (Förste et al 2005). The σ are formal for GGM02S, calibrated for EIGEN-CG03C. $\Delta\overline{C}_{\ell 0}$ are always larger than the linearly added sigmas.

ℓ	$\Delta\overline{C}_{\ell 0}$ (GGM02S-EIGEN-CG03C)	$\sigma_{\mathrm{X}} + \sigma_{\mathrm{Y}}$	f_ℓ (mas yr^{-1})
4	1.81×10^{-11}	3.7×10^{-12}	6.7
6	1.53×10^{-11}	1.8×10^{-12}	3.3
8	1.5×10^{-12}	1.1×10^{-12}	-
10	4.9×10^{-12}	8×10^{-13}	-
12	8×10^{-13}	7×10^{-13}	-
14	7.7×10^{-12}	6×10^{-13}	-
16	3.8×10^{-12}	5×10^{-13}	-
18	2.1×10^{-12}	5×10^{-13}	-
20	2.3×10^{-12}	4×10^{-13}	-
	total bias $\delta\mu = 22\%$		

Table 4: Bias due to the mismodelling in the even zonals of the models X=ITG-Grace03s (Mayer-Gürr 2007), based on GRACE-only accumulated normal equations from data out of September 2002-April 2007 (neither apriori information nor regularization used), and Y=GGM02S (Tapley et al 2005). The σ for both models are formal. $\Delta\overline{C}_{\ell 0}$ are always larger than the linearly added sigmas, apart from $\ell = 12$ and $\ell = 18$.

ℓ	$\Delta\overline{C}_{\ell 0}$ (ITG-Grace03s-GGM02S)	$\sigma_{\rm X} + \sigma_{\rm Y}$	f_ℓ (mas yr^{-1})
4	2.58×10^{-11}	8.6×10^{-12}	9.6
6	1.39×10^{-11}	4.7×10^{-12}	3.1
8	5.6×10^{-12}	2.9×10^{-12}	0.2
10	1.03×10^{-11}	2×10^{-12}	-
12	7×10^{-13}	1.8×10^{-12}	-
14	7.3×10^{-12}	1.6×10^{-12}	-
16	2.6×10^{-12}	1.6×10^{-12}	-
18	8×10^{-13}	1.6×10^{-12}	-
20	2.4×10^{-12}	1.6×10^{-12}	-
	total bias $\delta\mu = 27\%$		

Table 5: Bias due to the mismodelling in the even zonals of the models X = GGM02S (Tapley et al 2005) and Y = GGM03S (Tapley et al 2007) retrieved from data spanning January 2003 to December 2006. The σ for GGM03S are calibrated. $\Delta\overline{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, 6$. (The other zonals are of no concern)

ℓ	$\Delta\overline{C}_{\ell 0}$ (GGM02S-GGM03S)	$\sigma_{\mathrm{X}} + \sigma_{\mathrm{Y}}$	f_ℓ (mas yr^{-1})
4	1.87×10^{-11}	1.25×10^{-11}	6.9
6	1.96×10^{-11}	6.7×10^{-12}	4.2
8	3.8×10^{-12}	4.3×10^{-12}	0.1
10	8.9×10^{-12}	2.8×10^{-12}	0.1
12	6×10^{-13}	2.4×10^{-12}	-
14	6.6×10^{-12}	2.1×10^{-12}	-
16	2.1×10^{-12}	2.0×10^{-12}	-
18	1.8×10^{-12}	2.0×10^{-12}	-
20	2.2×10^{-12}	1.9×10^{-12}	-
	total bias $\delta\mu = 24\%$		

Table 6: Bias due to the mismodelling in the even zonals of the models X = EIGEN-GRACE02S (Reigber et al 2005) and Y = GGM03S (Tapley et al 2007). The σ for both models are calibrated. $\Delta \overline{C}_{\ell 0}$ are always larger than the linearly added sigmas apart from $\ell = 14, 18$.

ℓ	$\Delta \overline{C}_{\ell 0}$ (EIGEN-GRACE02S-GGM03S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr^{-1})
4	2.00×10^{-11}	8.1×10^{-12}	7.4
6	2.92×10^{-11}	4.3×10^{-12}	6.3
8	1.05×10^{-11}	3.0×10^{-12}	0.4
10	7.8×10^{-12}	2.9×10^{-12}	0.1
12	3.9×10^{-12}	1.8×10^{-12}	-
14	5×10^{-13}	1.7×10^{-12}	-
16	1.7×10^{-12}	1.4×10^{-12}	-
18	2×10^{-13}	1.4×10^{-12}	-
20	2.5×10^{-12}	1.4×10^{-12}	-
	total bias $\delta \mu = 30\%$		

Table 7: Bias due to the mismodelling in the even zonals of the models X = JEM01-RL03B, based on 49 months of GRACE-only data, and Y = GGM03S (Tapley et al 2007). The σ for GGM03S are calibrated. $\Delta \overline{C}_{\ell 0}$ are always larger than the linearly added sigmas apart from $\ell = 16$.

ℓ	$\Delta \overline{C}_{\ell 0}$ (JEM01-RL03B-GGM03S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr^{-1})
4	1.97×10^{-11}	4.3×10^{-12}	7.3
6	2.7×10^{-12}	2.3×10^{-12}	0.6
8	1.7×10^{-12}	1.6×10^{-12}	-
10	2.3×10^{-12}	8×10^{-13}	-
12	7×10^{-13}	7×10^{-13}	-
14	1.0×10^{-12}	6×10^{-13}	-
16	2×10^{-13}	5×10^{-13}	-
18	7×10^{-13}	5×10^{-13}	-
20	5×10^{-13}	4×10^{-13}	-
	total bias $\delta\mu = 17\%$		

Table 8: Bias due to the mismodelling in the even zonals of the models X = JEM01-RL03B and Y = ITG-Grace03s (Mayer-Gürr 2007). The σ for ITG-Grace03s are formal. $\Delta\overline{C}_{\ell 0}$ are always larger than the linearly added sigmas.

ℓ	$\Delta\overline{C}_{\ell 0}$ (JEM01-RL03B-ITG-Grace03s)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr^{-1})
4	2.68×10^{-11}	4×10^{-13}	9.9
6	3.0×10^{-12}	2×10^{-13}	0.6
8	3.4×10^{-12}	1×10^{-13}	0.1
10	3.6×10^{-12}	1×10^{-13}	-
12	6×10^{-13}	9×10^{-14}	-
14	1.7×10^{-12}	9×10^{-14}	-
16	4×10^{-13}	8×10^{-14}	-
18	4×10^{-13}	8×10^{-14}	-
20	7×10^{-13}	8×10^{-14}	-
	total bias $\delta\mu = 22\%$		

Table 9: Aliasing effect of the mismodelling in the even zonal harmonics estimated in the X=ITG-Grace03s (Mayer-Gürr 2007) and the Y=EIGEN-GRACE02S (Reigber et al 2005) models. The covariance matrix σ for ITG-Grace03s are formal, while the ones of EIGEN-GRACE02S are calibrated. $\Delta\overline{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, ..., 20$, apart from $\ell = 18$.

ℓ	$\Delta\overline{C}_{\ell 0}$ (ITG-Grace03s-EIGEN-GRACE02S)	$\sigma_X + \sigma_Y$	f_ℓ (mas yr^{-1})
4	2.72×10^{-11}	3.9×10^{-12}	10.1
6	2.35×10^{-11}	2.0×10^{-12}	5.1
8	1.23×10^{-11}	1.5×10^{-12}	0.4
10	9.2×10^{-12}	2.1×10^{-12}	0.1
12	4.1×10^{-12}	1.2×10^{-12}	-
14	5.8×10^{-12}	1.2×10^{-12}	-
16	3.4×10^{-12}	9×10^{-13}	-
18	5×10^{-13}	1.0×10^{-12}	-
20	1.8×10^{-12}	1.1×10^{-12}	-
	total bias $\delta\mu = 37\%$		

Table 10: Impact of the mismodelling in the even zonal harmonics estimated in the X=EGM2008 (Pavlis et al. 2008) and the Y=EIGEN-GRACE02S (Reigber et al 2005) models. The covariance matrix σ are calibrated for both EGM2008 and EIGEN-GRACE02S. $\Delta\overline{C}_{\ell 0}$ are larger than the linearly added sigmas for $\ell = 4, ..., 20$, apart from $\ell = 18$.

ℓ	$\Delta\overline{C}_{\ell 0}$ (EGM2008-EIGEN-GRACE02S)	$\sigma_{\rm X} + \sigma_{\rm Y}$	f_ℓ (mas yr^{-1})
4	2.71×10^{-11}	8.3×10^{-12}	10.0
6	2.35×10^{-11}	4.1×10^{-12}	5.0
8	1.23×10^{-11}	2.7×10^{-12}	0.4
10	9.2×10^{-12}	2.9×10^{-12}	0.1
12	4.1×10^{-12}	1.9×10^{-12}	-
14	5.8×10^{-12}	1.8×10^{-12}	-
16	3.4×10^{-12}	1.5×10^{-12}	-
18	5×10^{-13}	1.5×10^{-12}	-
20	1.8×10^{-12}	1.5×10^{-12}	-
	total bias $\delta\mu = 33\%$		

Table 11: Systematic percent error $\delta\mu$ in the measurement of the Lense-Thirring effect with LAGEOS, LAGEOS II and LARES according to eq. (10) and $\delta J_\ell = \Delta J_\ell$ up to degree $\ell = 60$ for the global Earth's gravity solutions considered here.

Models compared	$\delta\mu(\%)$
EGM2008−JEM01-RL03B	8%
EGM2008−GGM02S	27%
EGM2008−GGM03S	5%
EGM2008−ITG-Grace02	4%
EGM2008−ITG-Grace03	0.1%
EGM2008−EIGEN-CG03C	38%
EGM2008−EIGEN-GRACE02S	53%
JEM01-RL03B−GGM02S	28%
JEM01-RL03B−GGM03S	10%
JEM01-RL03B−ITG-Grace02	8%
JEM01-RL03B−ITG-Grace03s	8%
JEM01-RL03B−EIGEN-CG03C	44%
JEM01-RL03B−EIGEN-GRACE02S	57%
GGM02S−GGM03S	24%
GGM02S−ITG-Grace02	28%
GGM02S−ITG-Grace03s	26%
GGM02S−EIGEN-CG03C	27%
GGM02S−EIGEN-GRACE02S	36%
GGM03S−ITG-Grace02	5%
GGM03S−ITG-Grace03s	5%
GGM03S−EIGEN-CG03C	36%
GGM03S−EIGEN-GRACE02S	52%
ITG-Grace02−ITG-Grace03s	4%
ITG-Grace02−EIGEN-CG03C	39%
ITG-Grace02−EIGEN-GRACE02S	54%
ITG-Grace03s−EIGEN-CG03C	38%
ITG-Grace03s−EIGEN-GRACE02S	53%
EIGEN-CG03C−EIGEN-GRACE02S	27%

It must be stressed that they may be still optimistic: indeed, computations for $\ell > 60$ become unreliable because of numerical instability of the results (obtained with two different softwares).

If, instead, one assumes $\delta J_\ell = s_\ell$, $\ell = 2, 4, 6, \ldots$ i.e., the standard deviations of the sets of all the best estimates of J_ℓ for the models considered here the systematic bias, up to $\ell = 60$, amounts to 26%. Again, also this result may turn out to be optimistic for the same reasons as before.

It must be pointed out that the evaluations presented here rely upon calculations of the coefficients $\dot{\Omega}_{.\ell}$ performed with the well known standard approach by Kaula Kaula (1966); it would be important to try to follow also different computational strategies in order to test them.

It is worthwhile noting that also the impact of the subtle non-gravitational perturbations will be different with respect to the original proposal because LARES will fly in a different and lower orbit and its thermal behavior will probably be different with respect to LAGEOS and LAGEOS II. The reduction of the impact of the thermal accelerations, like the Yarkowsky-Schach effects, should have been reached with two concentric spheres. However, as explained in Andrés (2007), this solution will increase the floating potential of LARES because of the much higher electrical resistivity and, thus, the perturbative effects produced by the charged particle drag. Moreover, drag will increase also because of the lower orbit of the satellite, both in its neutral and charged components. Also the Earth's albedo, with its anisotropic components, should have a major effect.

Another point which must be considered is the realistic orbit accuracy obtainable for LARES. Indeed, at a lower orbit the normal points RMS will be probably higher with respect to the present RMS obtained for the two LAGEOS satellites (a few mm), as we presently know for the Stella and Starlette normal points. Of course, such an accuracy is a

function of several aspects.

4. Conclusions

In this Chapter we have shown how the so far published evaluations of the total systematic error in the Lense-Thirring measurement with the combined nodes of the SLR LAGEOS and LAGEOS II satellites due to the classical node precessions induced by the even zonal harmonics of the geopotential are optimistic. Indeed, they are all based on the use of the covariance matrix's sigmas, more or less reliably calibrated, of various Earth gravity model solutions used one at a time separately in such a way that the model X yields an error of $x\%$, the model Y yields an error $y\%$, etc. Instead, comparing the estimated values of the even zonals for different pairs of models allows for a much more realistic evaluation of the real uncertainties in our knowledge of the static part of the geopotential. As a consequence, the bias in the Lense-Thirring effect measurement is about $3 - 4$ times larger than that so far claimed, amounting to various tens percent (37% for the pair EIGEN-GRACE02S and ITG-GRACE03s, about $25 - 30\%$ for the other most recent GRACE-based solutions).

Applying the same strategy to the ongoing LARES mission shows that the goal of reaching a 1% measurement of the Lense-Thirring effect with LAGEOS, LAGEOS II and LARES is optimistic. Indeed, since LARES will orbit at a lower altitude with respect to the LAGEOS satellites, a larger number of even zonal harmonics are to be taken into account. Assessing realistically their impact is not easy. Straightforward calculations up to degree $\ell = 60$ with the standard Kaula's approach yield errors as large as some tens percent. Such an important point certainly deserves great attention. Another issue which may potentially degrade the expected accuracy is the impact of some non-gravitational perturbations which would have a larger effect on LARES than expected because of its lower orbit.

REFERENCES

Andrés J I 2007, Enhanced Modelling of LAGEOS Non-Gravitational Perturbations. PhD Thesis book. (Ed. Sieca Repro Turbineweg, 20, 2627, BP Delft, The Netherlands). ISBN 978-90-5623-081-4.

Ciufolini I 1986 *Phys. Rev. Lett.* **56** 278

Ciufolini I 1996 *N. Cim. A* **109** 1709

Ciufolini I 2004 LARES/WEBER-SAT, frame-dragging and fundamental physics (*Preprint* gr-qc/0412001v3)

Ciufolini I 2006 On the orbit of the LARES satellite (*Preprint* gr-qc/0609081v1)

Ciufolini I 2008 downloadable at http://www.infn.it/indexen.php→ASTROPARTICLE PHYSICS→Calendario riunioni→Roma, 30 gennaio 2008→14:30 Aggiornamento LARES (20')→lares_dellagnello.pdf, pag 17

Ciufolini I and Pavlis E C 2004 *Nature* **431**(21 October (2004) 958

Ciufolini I and Pavlis E C 2005 *New Astron.* **10** 636

Ciufolini I, Lucchesi D M, Vespe F and Mandiello A 1996 *N. Cim. A* **109** 575

Ciufolini I et al 1998a *LARES Phase A* (Rome: University La Sapienza)

Ciufolini I, Pavlis E C, Chieppa F, Fernandes-Vieira E and Pérez-Mercader J 1998b *Science* **279** 2100

Ciufolini I, Pavlis E C and Peron R 2006 *New Astron.* **11** 527

Cugusi L and Proverbio E 1978 *Astron. Astrophys.* **69** 321

Everitt C W F 1974 The Gyroscope Experiment I. General Description and Analysis of Gyroscope Performance in *Proc. Int. School Phys. "Enrico Fermi" Course LVI* ed B Bertotti (New York: New Academic Press) pp 331–60

Everitt C W F et al 2001 Gravity Probe B: Countdown to Launch in *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space* ed C Lämmerzahl, C W F Everitt and F W Hehl (Berlin: Springer) pp 52–82

Inversi P and Vespe F 1994 *Adv. Space Res.* **14** 73

Iorio L 2002 *Class. Quantum Grav.* **19** L175

Iorio L 2003 *Celest. Mech. Dyn. Astron.* **86** 277

Iorio L 2005a *New Astron.* **10** 616

Iorio L 2005b *J. Cosmol. Astropart. Phys.* JCAP07 (2005)008

Iorio L 2006a *J. Geod.* **80** 128

Iorio L 2006b *Gen. Relativ. Gravit.* **38** 523

Iorio L 2007a *Planet. Space Sci.* **55** 1198

Iorio L 2007b *Europhys. Lett.* **80** 40007

Iorio L 2007c *Planet. Space Sci.* **55** 503

Iorio L (ed) 2007d *The Measurement of Gravitomagnetism: A Challenging Enterprise* (Hauppauge: NOVA)

Iorio L and Morea A 2004 *Gen. Relativ. Gravit.* **36** 1321

Iorio L and Doornbos E 2005 *Gen. Relativ. Gravit.* **37** 1059

Iorio L, Lucchesi D M and Ciufolini I 2002 *Class. Quantum Grav.* **(19** 4311

Förste C et al 2005 A New High Resolution Global Gravity Field Model Derived From Combination of GRACE and CHAMP Mission and Altimetry/Gravimetry Surface Gravity Data *EGU General Assembly 2005 (Vienna, AT, 24–29 April 2005)*

Kaula W M 1966 *Theory of Satellite Geodesy* (Waltham: Blaisdell)

Lemoine F G et al 1998 *NASA/TP-1998-206861*

Lense J and Thirring H 1918 *Phys. Z.* **19** 156

Lerch F J, Nerem R S, Putney B H, Felsentreger T L, Sanchez B V, Marshall J A, Klosko S M, Patel G B, Williamson R G and Chinn D S 1994 *J. Geophys. Res.* **99**(B2) 2815

Lucchesi D M 2001 *Planet. Space Sci.* **49** 447

Lucchesi D M 2002 *Planet. Space Sci.* **50** 1067

Lucchesi D M 2003 *Geophys. Res. Lett.* **30** 1957

Lucchesi D M 2004 *Celest. Mech. Dyn. Astron.* **88** 269

Lucchesi D M 2005 *Int. J. Mod. Phys. D* **14** 1989

Lucchesi D M 2007 *Adv. Sp. Res.* **39** 1559

Lucchesi D M and Paolozzi A 2001 A cost effective approach for LARES satellite *XVI Congresso Nazionale AIDAA (Palermo, IT, 24-28 September 2001)*

Lucchesi D M, Ciufolini I, Andrés J I, Pavlis E C, Peron R, Noomen R and Currie D G 2004 *Planet. Space Sci.* **52** 699

Mashhoon B 2007 Gravitoelectromagnetism: A Brief Review *The Measurement of Gravitomagnetism: A Challenging Enterprise* ed L Iorio (Hauppauge: NOVA) pp. 29–39

Mayer-Gürr T, Eicker A and Ilk K-H 2006 ITG-GRACE02s: a GRACE gravity field derived from short arcs of the satellite's orbit *1st Int. Symp. of the International Gravity Field Service "GRAVITY FIELD OF THE EARTH"(Istanbul, TR, 28 August–1 September 2006)*

Mayer-Gürr T 2007 ITG-Grace03s: The latest GRACE gravity field solution computed in Bonn *Joint Int. GSTM and DFG SPP Symp. (Potsdam, D, 15–17 October 2007)* (http://www.geod.uni-bonn.de/itg-grace03.html)

Milani A, Nobili A M and Farinella P 1987 *Non-gravitational perturbations and satellite geodesy* (Bristol: Adam Hilger)

Pavlis E C 2002 Geodetic contributions to gravitational experiments in space *Recent Developments in General Relativity: Proc. 14th SIGRAV Conf. on General Relativity and Gravitational Physics (Genova, IT, 18–22 September 2000)* ed R Cianci, R Collina, M Francaviglia and P Fré (Milano: Springer) pp 217–33

Pavlis N K et al. 2008 Paper presented at the 2008 General Assembly of the European Geosciences Union, Vienna, Austria, April 13-18, 2008.

Pfister H 2007 *Gen. Relativ. Gravit.* **39** 1735

Pugh G E 1959 *WSEG Research Memorandum No. 11*

Reigber Ch, Schmidt R, Flechtner F, König R, Meyer U, Neumayer K-H, Schwintzer P and Zhu S Y 2005 *J. Geodyn.* **39** 1

Ries J C, Eanes R J, Watkins M M and Tapley B 1989 Joint NASA/ASI Study on Measuring the Lense-Thirring Precession Using a Second LAGEOS Satellite *CSR-89-3* (The University of Texas at Austin: Center for Space Research)

Ries J C, Eanes R J and Tapley B D 2003a Lense-Thirring Precession Determination from Laser Ranging to Artificial Satellites *Nonlinear Gravitodynamics. The Lense–Thirring Effect* ed R J Ruffini and C Sigismondi (Singapore: World Scientific) pp 201–11

Ries J C, Eanes R J, Tapley B D and Peterson G E 2003b Prospects for an Improved Lense-Thirring Test with SLR and the GRACE Gravity Mission *Proc. 13th Int. Laser Ranging Workshop, NASA CP (2003-212248* ed R Noomen, S Klosko, C Noll and M Pearlman (Greenbelt: NASA Goddard) (*Preprint* http://cddisa.gsfc.nasa.gov/lw13/lw_proceedings.html#science)

Ruggiero M L and Tartaglia A 2002 *N. Cim. B* **117** 743

Schäfer G 2004 *Gen. Relativ. Gravit.* **36** 2223

Schiff L 1960 *Phys. Rev. Lett.* **4** 215

Tapley B D et al 2005 *J. Geod.* **79** 467

Tapley B D, Ries J, Bettadpur S, Chambers D, Cheng M, Condi F, Poole, S 2007 American Geophysical Union, Fall Meeting 2007, abstract #G42A-03

Van Patten R A and Everitt C W F 1976a *Phys. Rev. Lett.* **36** 629

Van Patten R A and Everitt C W F 1976b *Celest. Mech. Dyn. Astron.* **13** 429

Vespe F 1999 *Adv. Space Res.* **23** 699

Vespe F and Rutigliano P 2005 *Adv. Space Res.* **36** 472